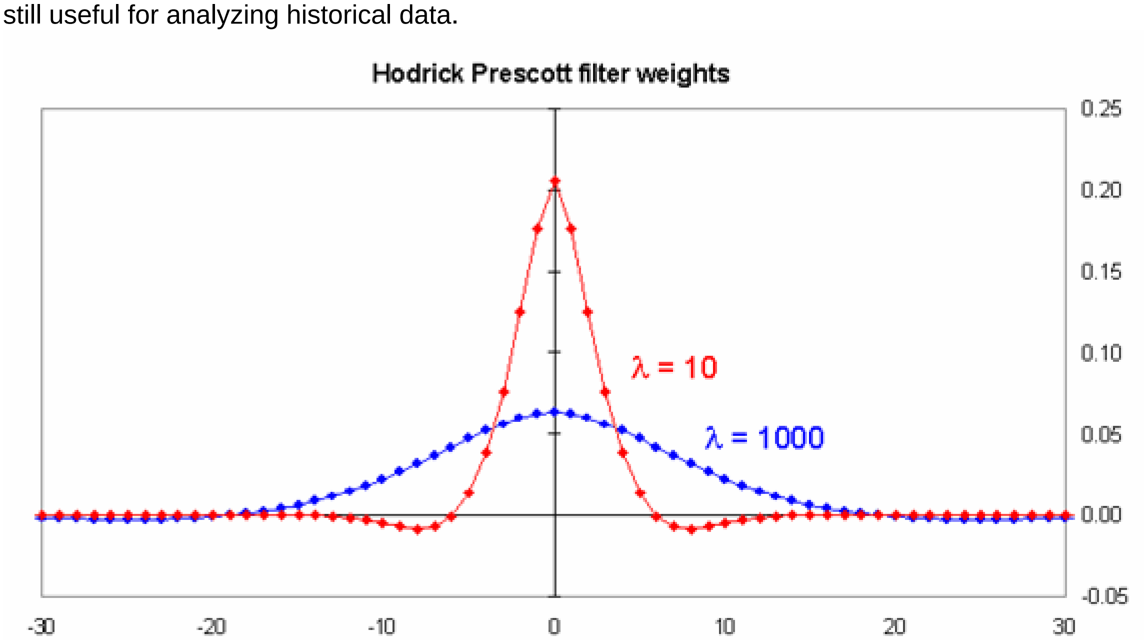
Another look at the Hodrick Prescott Trend Filter
Dave Gransden

The variance of a signal is the sum of the squares of the errors, error being the difference between the signal and the original data. We can define the roughness of a signal to be the sum of the squares of the second derivative* of that signal, a straight line is of course perfectly smooth and its second derivative is zero.

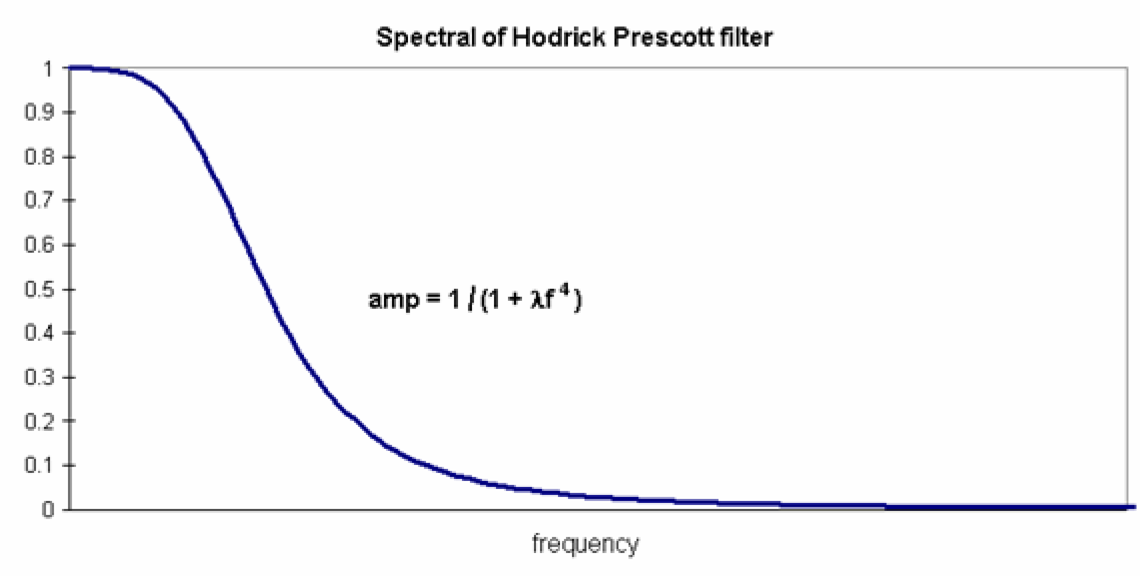
To define the HP trend we multiply the roughness by a chosen constant “lambda” and add the variance. For a given value of lambda, if this sum is minimized one can say that the trend is optimum; it follows the data as closely as possible while being as smooth as possible. A larger value of lambda makes the trend smoother or straighter, while a smaller value makes it follow the data closer, until at lambda =0 it simply equals the data.

Although this trend definition is elegant the solution seems rather complex. H.Kim has shown that the solution to the HP minimization equation can be found with matrix operations. In fact the matrix is of a type** that makes a matrix multiply a convolution, which is another fancy way of saying a weighted average.

A graph of the HP weights for two values of lambda is shown. When lambda is large the curve is wider; more points are included in the average. Note that the sum of all the weights is one. The weights go slightly negative on both sides but that has little effect on the resulting trend. The weights are symmetrical into the future as well as the past making it unsuitable for trading, but still useful for analyzing historical data.



A weighted average can also be analyzed by looking at its frequency spectrum. The spectral graph shows that the HP filter is a simple, smooth, low pass filter. Higher frequencies have their amplitude reduced proportional to the 4th power of the frequency.



* The second derivative of sampled data is simply (twice today's value) minus (yesterday plus tomorrow).

** A circulant matrix is one in which each row is a shifted copy of the row above, its inverse will also be circulant.